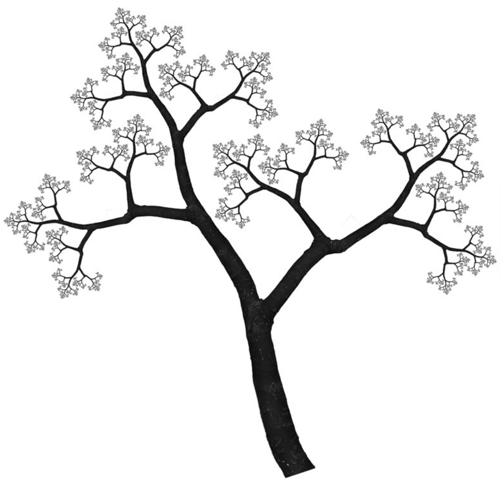
Fractal Trees

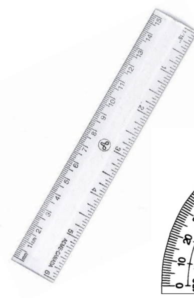
Teacher's Intro

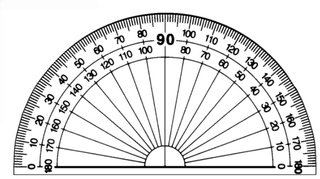
OVERVIEW

In this lesson, we will use the fractal branching of a tree to explore some of the basics of fractions and ratios in a simple, tangible way. And, we'll use a protractor to see how simple a complicated tree can be.

Like all fractals, a tree grows by repeating a simple process: Branching. This means a little piece of a tree has the same general shape as a bigger part, which also is similar to the entire tree. This property of fractals is known as "self-similarity." In this activity, we will verify the similarity of branches of a tree to the whole object. To do so, we will compare the lengths of connected branches, and see that even as they get smaller, the ratio of the lengths of branches stays the same. We'll also use a protractor to compare the angles of the branches.







NM Math Standards:

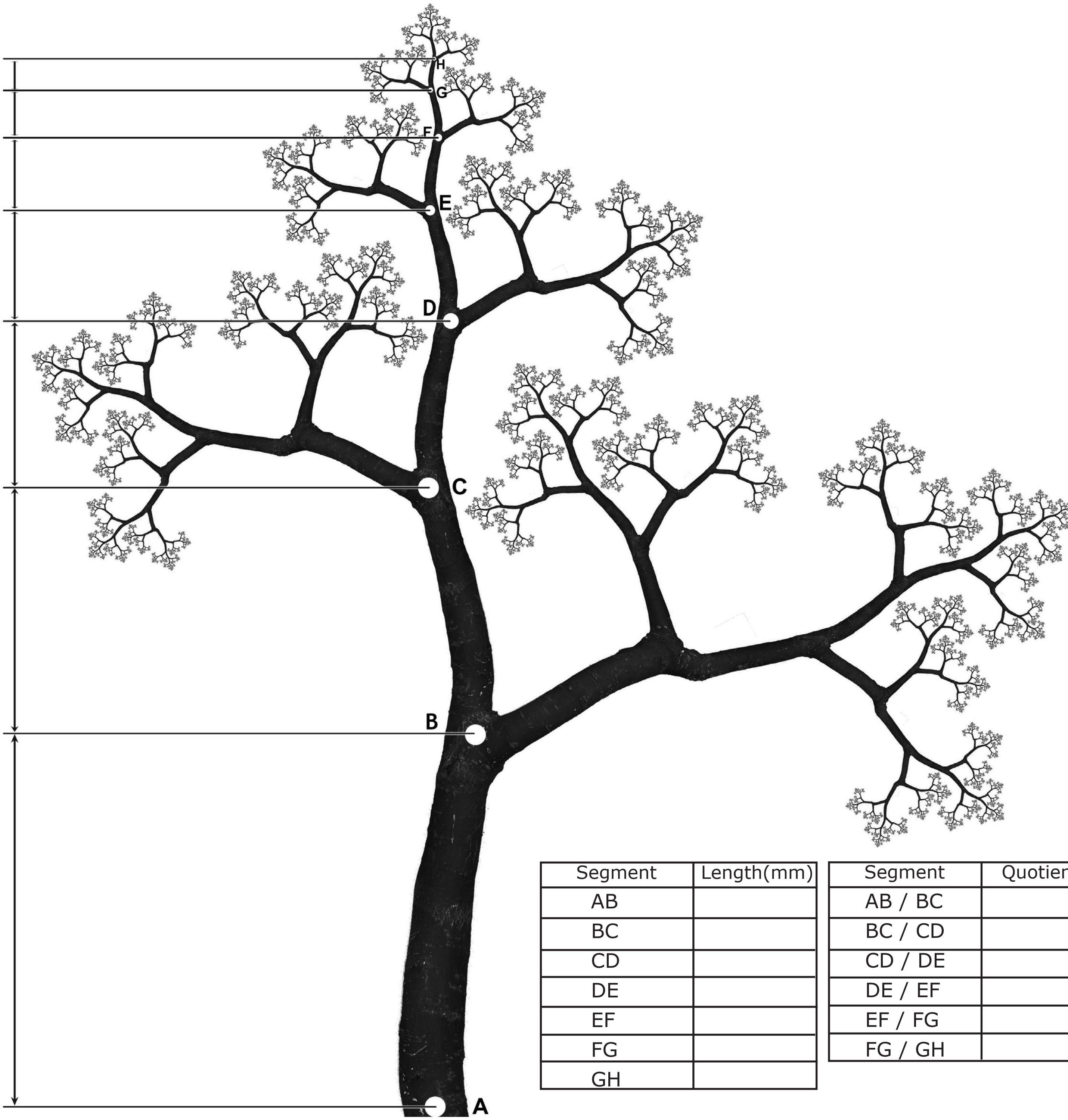
K.G.4.2 Find and describe geometric shapes in nature or architecture.
(K,1,2).A.1.3 Recognize, reproduce, describe, extend, and create repeating patterns.
2.D.1.1 Collect numerical data systematically.
2.G.4.4 Relate geometric ideas to numbers
3.M.2.4 Recognize a 90-degree angle and use it as a strategy to estimate the size of other angles.
4.M.2.6 Use tools to measure angles (e.g., protractor, compass).
4.G.1.1 Identify and compare congruent and similar figures
5.D.1.1 Construct, read, analyze, and interpret tables, charts, etc.
5.M.1.1 Understand properties (e.g., length, area, weight, volume) and select the appropriate type of unit for measuring each using both U.S. customary and metric systems.
6.A.4.3 Use ratios to predict changes in proportional situations.
6.N.3.7 Compute and perform multiplication and division of fractions and decimals and apply these procedures to solving problems.
6.N.3.6 Interpret and use ratios in different contexts.

NM Science Standards:

Recognize the differences between mature and immature plants and animals (e.g. trees and seedlings (1)
Measure length with appropriate tools and express those measurements in accurate mathematical language (2)
Make predictions based on observed patterns as opposed to random guessing (2)
Use numerical data in describing and comparing objects(3)
Collect data in an investigation and analyze those data (3)
Use appropriate units to make precise and varied measurements (5)

Fractal Trees

Trees are *fractals*, which means we see the same shape repeating at different sizes. This fractal tree grows from the bottom up by branching over and over again. In this fractal tree, pieces of the tree at different sizes have the same shapes, patterns and relationships. We'll test this by measuring and comparing the distances between branching points, and the angles between the branches.



Segment	Length(mm)
AB	
BC	
CD	
DE	
EF	
FG	
GH	

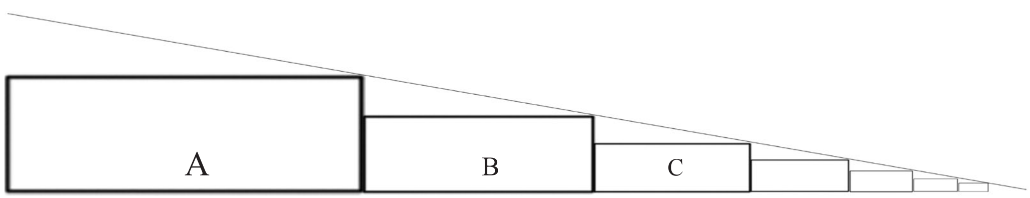
Segment	Quotient
AB / BC	
BC / CD	
CD / DE	
DE / EF	
EF / FG	
FG / GH	

On the previous page, use a ruler to measure the distances between the bottom of the tree (marked 'A') and the first branching point, (B). Measure the distance in millimeters, and record your measurement in the table. Next, measure the distance between point 'B' and point 'C', and record the value in the table. Continue measuring and fill in the remaining distances between branches in the table.

In fractals, we find the same patterns repeating again and again at smaller sizes. Next, we want to see how the size between branches compares from one branch to the next. To do so, we'll calculate the *Quotient* between the lengths of the branches. This is easy to do, but will require a calculator. All we have to do is divide the length of one branch, say "AB" by the length of the next smaller branch, "BC". The quotient tells us how much bigger the branch is than the next smaller branch. So if "AB" were twice as long as "BC", the quotient would be 2 (or we could say the ratio was 2:1, that is "Two to One"). If "AB" were only one and a half times bigger than "BC", the quotient would be 1.5, and the ratio would be 1.5:1, or 3:2. If it were one and a quarter (1.25) times bigger than the next, the ratio would be 1.25:1. Get it? Use your calculator to compute the rest of the quotients in the table.

What pattern do you see? Are the relationships the same from one branch to the next? Are they close?

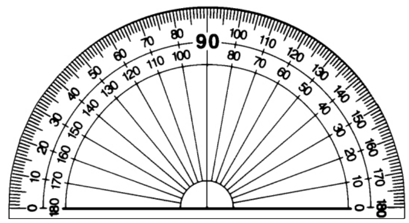
Do you see why it's easier to measure the branch distances in millimeters instead of inches?

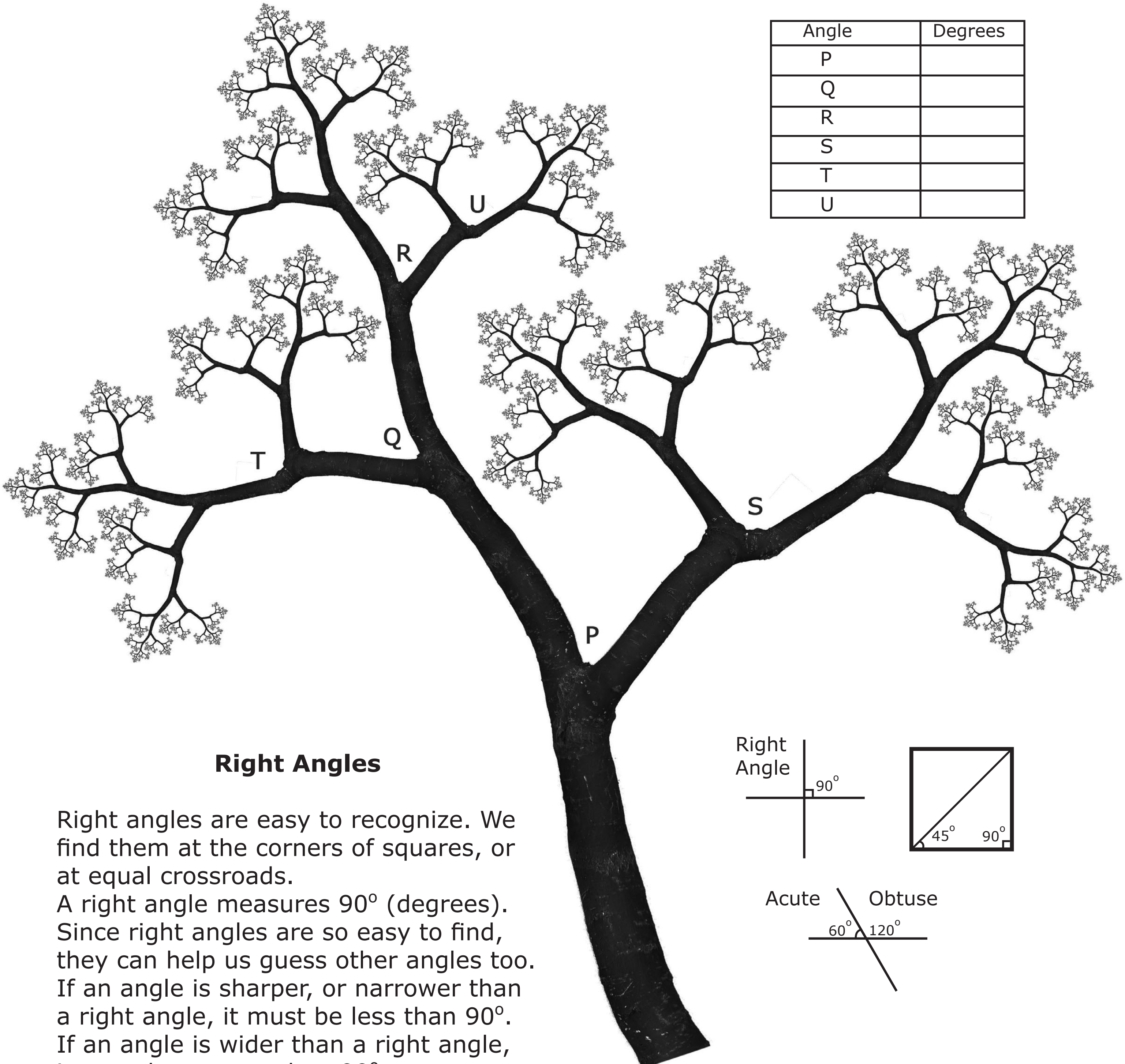


The Ratio of the size of the boxes is the same - each one is the same amount larger than the one before. The relationship of A to B is the same as that of B to C, etc.

Next, we'll use a protractor to measure the angles between the branches. On the next page, measure the angles and fill in the values in the table.

What do you notice about the various angles in the tree? How many distinct angles are there? (Not how many total branching points, but how many kinds of angles can you find?)





Angle	Degrees
P	
Q	
R	
S	
T	
U	

Right Angles

Right angles are easy to recognize. We find them at the corners of squares, or at equal crossroads.
A right angle measures 90^o (degrees).
Since right angles are so easy to find, they can help us guess other angles too.
If an angle is sharper, or narrower than a right angle, it must be less than 90^o.
If an angle is wider than a right angle, it must be greater than 90^o.

Before you even use a protractor to measure the angles of the branches, compare angles “P” and “S” to a right angle. Are they greater or less than 90^o? Label angles “P” and “S” “Acute” or “Obtuse”.